Year Six Fraction Understanding: A Part of the Whole Story

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A range of assessment tasks was developed for use in one-to-one interviews in December 2005 with 323 Grade 6 students in Victoria. In this paper, we summarise briefly the research literature on fractions, describe the process of development of assessment tasks, share data on student achievement on these tasks, and suggest implications for curriculum and classroom practice. Particular emphasis in the discussion is given to students' judgements and strategies in comparing fractions. A particular feature of this report is that one-to-one interview assessment data were collected from a larger number of students than is typically the case in these kinds of studies. Recommendations arising from these data include the importance of teachers understanding and presenting a wider range of sub-constructs of fractions to students in both teaching and assessment than is currently the case, using a greater variety of models, and taking available opportunities to use the interview tasks with their own students.

Theoretical Background

Fractions are widely agreed to form an important part of middle years mathematics curriculum (Lamon, 1999; Litwiller & Bright, 2002), underpinning the development of proportional reasoning, and important for later topics in mathematics, including algebra and probability. However, it is clear that it is a topic which many teachers find difficult to understand and teach (Post, Cramer, Behr, Lesh, & Harel, 1993), and many students find difficult to learn (Behr, Lesh, Post, & Silver, 1983; Kieren, 1976; Streefland, 1991). Among the factors that make rational numbers in general, and fractions in particular difficult to understand are their many representations and interpretations (Kilpatrick, Swafford, & Findell, 2001).

There is considerable evidence that the difficulties with fractions are greatly reduced if instructional practices involve providing students with the opportunity to build concepts as they are engaged in mathematical activities that promote understanding (Bulgar, Schorr, & Maher, 2002; Olive, 2001).

In the Early Numeracy Research Project (Clarke, et al., 2002), a task-based, interactive, one-to-one assessment interview was developed, for use with students in the early years of schooling. This interview was used with over 11 000 students, aged 4 to 8, in 70 Victorian schools at the beginning and end of the school year, thus providing high quality data on what students knew and could do in these early grades, across the mathematical domains of Number, Measurement, and Geometry. There was equal emphasis in the teachers' record of interview on *answers* and the *strategies* that led to these answers.

The use of a student assessment interview, embedded within an extensive and appropriate inservice or preservice program, can be a powerful tool for teacher professional learning, enhancing teachers' knowledge of how mathematics learning develops and knowledge of individual mathematical understanding, as well as content knowledge and pedagogical content knowledge (Clarke, Mitchell, & Roche, 2005; Schorr, 2001).

The success of the interview and comments from middle years' teachers prompted the authors to consider extending the use of the assessment interview to the middle years of schooling (Grades 5 to 8). As a first, major step in this process, it was decided to focus the interview on the important mathematical topics of fractions and decimals. This paper reports the process and findings from this work, with particular emphasis on fractions.

Fractions: Constructs and Models

Much of the confusion in teaching and learning fractions appears to arise from the many different interpretations (constructs) and representations (models). Also, generalisations that have occurred during instruction on whole numbers have been misapplied to fractions (Streefland, 1991). Finally, there appears to be a void between student conceptual and procedural understanding of fractions and being able to link intuitive knowledge (or familiar contexts) with symbols (or formal classroom instruction) (Hasemann, 1981; Mack, 2002). The dilemma for both teachers and students is how to make all the appropriate connections so that a mature, holistic, and flexible understanding of fractions and the wider domain of rational numbers can be obtained.

Kieren (1976) was able to identify several different interpretations (or constructs) of *rational numbers* and these are often summarised as part-whole, measure, quotient (division), operator, ratio, and decimals. For the purpose of this review these interpretations are explained in the context of fractions.

The *part-whole* interpretation depends on the ability to partition either a continuous quantity (including area, length, and volume models) or a set of discrete objects into equal sized subparts or sets. The part-whole construct is the most common interpretation of fractions and likely to be the first interpretation that students meet at school. Lamon (2001) suggested that "mathematically and psychologically, the part-whole interpretation of fraction is not sufficient as a foundation for the system of rational numbers" (p. 150).

A fraction can represent a *measure* of a quantity relative to one unit of that quantity. Lamon (1999) explained that the measure interpretation is different from the other constructs in that the number of equal parts in a unit can vary depending on how many times you partition. This successive partitioning allows you to "measure" with precision. We speak of these measurements as "points" and the number line provides a model to demonstrate this.

A fraction (a/b) may also represent the operation of *division* or the result of a division such that 3÷5 = 3/5. The division interpretation may be understood through partitioning and equal sharing. These two activities have been the focus of much research (Empson, 2003).

A fraction can be used as an *operator* to shrink and stretch a number such as 3/4 x 12 = 9 and 5/4 x 8 = 10. The misconception that multiplication always makes bigger and division always makes smaller is common (Bell, Fischbein, & Greer, 1984). It could also be suggested that student lack of experience with using fractions as operators may also contribute to this misconception.

Fractions can be used as a method of comparing the sizes of two sets or two measurements such as "the number of girls in the class is 3/5 the number of boys", i.e., a *ratio*. Post et al. (1993) claim "ratio, measure and operator constructs are not given nearly enough emphasis in the school curriculum" (p. 328).

Although these constructs can be considered separately they have some unifying elements or "big ideas". Carpenter, Fennema, and Romberg (1993) identified three unifying elements to these interpretations and they are: identification of the unit, partitioning, and the notion of quantity.

Method

Focusing on the rational number constructs of part-whole, measure, division, and operator, and the "big ideas" of the unit, using discrete and continuous models, partitioning, and the relative size of fractions, a range of around 50 assessment tasks was established, drawing upon tasks that had been reported in the literature, and supplemented with tasks that the research team developed. These tasks were piloted with around 30 students in Grades 4 to 9, refined, and piloted again (Mitchell & Clarke, 2004).

Using a selection of the set of tasks, 323 Grade 6 students were interviewed at the end of the school year. The schools and students were chosen to be broadly representative of Victorian students, on variables such as school size, location, proportion of students from non-English speaking backgrounds, and socio-economic status. A team of ten interviewers, all experienced primary teachers, with at least 4 years' experience in one-to-one assessment interviews of this kind, participated in a day's training on the use of the interview tasks, including viewing sample interviews on video.

The tasks were administered individually over a 30- to 40-minute period in the students' own schools, with interviews following a strict script for consistency, and using a standard record sheet to record students' answers, methods and any written calculations or sketches. Each actual response to a question was given a code by the authors, and a trained team of coders took the data from the record sheets, coded each response, and entered it into SPSS. Key findings are provided in the following section.

Results

In this section, data from the 323 Grade 6 students are provided on eight of the tasks, organised around relevant sub-constructs of fractions (Kieren, 1976). In each case, the task is outlined, the mathematical idea it was designed to address is stated, the percentage student success rate is given, and common strategies and solutions, including misconceptions, are outlined.

Part-whole

Three tasks focused on part-whole thinking.

1. *Fraction Pie task* (adapted from Cramer, Behr, Post, & Lesh, 1997). Students were shown the pie model (Figure 1), and asked:
a) What fraction of the circle is part B?
b) What fraction of the circle is part D?

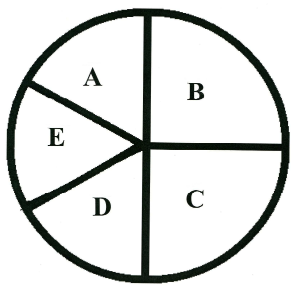

Figure 1. Fraction Pie task.

Part (a) was relatively straightforward, with 83.0% of students answering 1/4. Of the total group, 3.3% offered a correct equivalent fraction, decimal, or percentage answer, whereas 5.6% and 1.9% answered “1/5” and “1/2”, respectively. Part (b) was more difficult, with only 42.7% giving a correct answer, with 13.6% answering 1/5 (presumably based on “five parts”). The same percentage answered 1/3, probably focusing only on the left-hand side.

2. *Dots Array task*. Students were shown the array in Figure 2, and asked, “what fraction of the dots is black?” They were then asked to state “another name for that fraction”; 76.9% gave a correct answer, with the three most common answers being 2/3 (35.6%), 12/18 (30.7%), and 4/6 (8.7%). The most common error was 3/4. Only 53.5% of students were able to offer another correct name for the fraction, with 4/6 being the most common response (17.0%). These data indicate that students generally showed a flexible approach to *unitising* (Lamon, 1999).

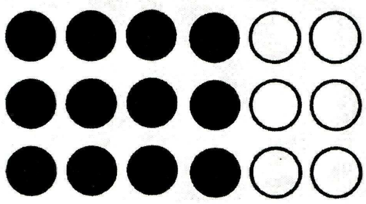

Figure 2. Dots Array task.

3. *Draw me a whole task (part a).* In assessing students’ capacity to move from the part to the whole, acknowledged by Lamon (1999) and others as an important skill, students were shown a rectangle (shaded grey in Figure 3), and asked, “if this is two-thirds of a shape, please draw the whole shape,” while explaining their thinking. 64.1% were able to do so successfully, with 28.5% of them dividing the original shape into two equal parts first, and 35.6% showing no visible divisions.

Figure 3. A student’s correct solution for Draw me a whole (part a).

Draw me a whole task (part b). Students were presented with a different rectangle (shaded part in Figure 4), told that it was “four thirds,” and asked to show the whole. In this case, 40.5% drew a correct shape, with just under half of these breaking the original rectangle into four parts, indicating three of these as the whole.

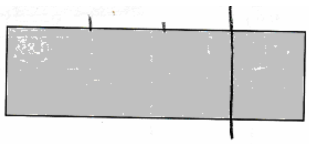

Figure 4. A student’s correct solution for Draw me a whole (part b).

Fraction as an Operator

4. *Simple operators.* Students were posed four questions, with no visual prompt, which required students to work out the answer in their heads. They were as follows: “… one-half of six?” (97.2% success); “… one-fifth of ten?” (73.4%); “… two-thirds of nine? (69.7%); and “… one third of a half?” (17.6%). The data on the last item, by

far the most difficult of this set, are interesting in light of the relative difficulty with the related pie task.

Fractions as Measure

5. *Number line (parts a, b & c).* Students were asked to "please draw a number line and put two thirds on it". If students did not choose to indicate where 0 and 1 should be in their drawing, they were asked by the interviewer, "where does zero go? … where does 1 go?" Only 51.1% of students were successful in correctly locating 2/3 on the number line. A common error was placing 2/3 after 1 (see Figure 5), or two-thirds along some line, e.g., at 4 on a number line from 0 to 6, or two-thirds of the way from 0 to 100 (see Figure 6).

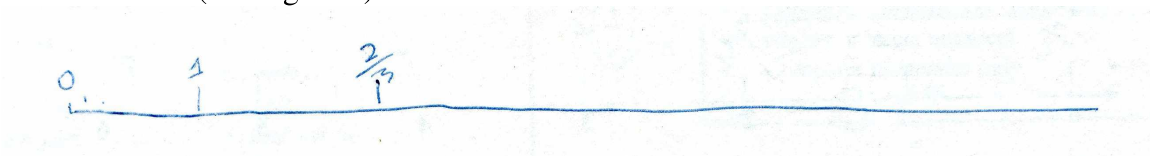

Figure 5. A student's incorrect solution for placing 2/3 on a number line (part a).

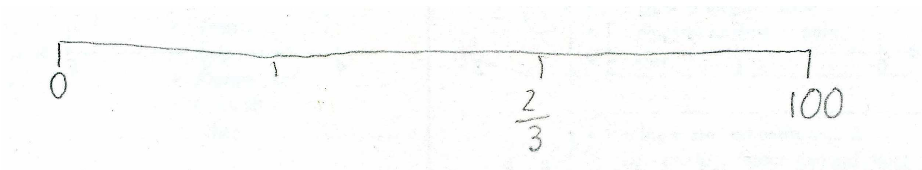

Figure 6. Another student's incorrect solution for placing 2/3 on a number line (part a).

Given a number line as shown in Figure 7, students were then asked to mark, in turn, six thirds (part b) and eleven sixths (part c) (Baturo & Cooper, 1999). Only 32.8% and 25.4% were successful, respectively. Many placed 6/3 on 6 or 3. Several students located 11/6 well to the right of 6.

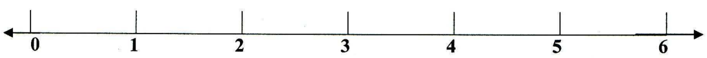

Figure 7. Number line task (part b & c).

6. *Construct a Sum.* In a task designed to get at students' understanding of the "size" of fractions, we used the Construct a Sum task (Behr, Wachsmuth, & Post, 1985). The student is directed to place number cards in the boxes to make fractions so that when you add them the answer is as close to one as possible, but not equal to one. The number cards included 1, 3, 4, 5, 6, and 7 (Figure 8). Each card could be used only once. The capacity for students to move cards around as they consider possibilities is a strong feature of this task. Only 25.4% of students produced a solution within 0.1 of 1, the most common response being 1/5 + 3/4 (5.3% of the total group). 24.5% of students chose fractions at least 0.5 away from 1, and most of these included an improper fraction. The answer closest to one (1/7 + 5/6) was chosen by only four students.

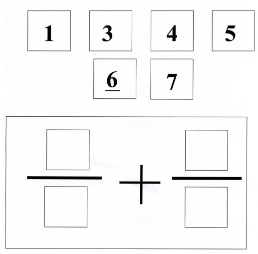

Figure 8. Construct a Sum task.

7. *Fraction pairs task.* Another task that we used to assess the important notion of fraction as a quantity is the *fraction pairs task*. Eight fraction pairs were shown to students, one pair at a time (see Figure 9). Each pair, typed on a card, was placed in front of the student, and the student was asked to point to the larger fraction of the pair and explain their reasoning. No opportunity was given for the students to write or draw anything. Our interest was in mental strategies.

a)	3/8	7/8	e)	2/4	4/2
b)	1/2	5/8	f)	3/7	5/8
c)	4/7	4/5	g)	5/6	7/8
d)	2/4	4/8	h)	3/4	7/9

Figure 9. The eight fraction pairs used in the study.

The intention was that, based on previous piloting, the tasks were presented in order of increasing difficulty. This proved not always to be the case.

For each task, the interviewer circled the student's chosen fraction on the interview record sheet, and recorded the student's reasons, choosing from a list of common explanations. For example, the choices given for the pair 3/4 and 7/9 were:

- Residual with equivalent (2/8 > 2/9)
- Residual thinking (1/4 > 2/9) with proof
- Converts to decimals
- Common denominator
- Higher or larger numbers
- Other ………………………….

If the method offered by the student did not correspond to any of the listed strategies, the interviewer noted the method used under "Other", making every effort to record all the words used by the student in the explanation.

Data analysis involved determining the percentage of students who gave the correct answer, and then for both correct and incorrect choices, the percentage of students who used each particular strategy. The list of strategies was expanded during data analysis to incorporate any strategies which were common, from the "Other" category.

Table 1 shows the percentage of students who selected the appropriate fraction from the pair (or indicated both were equal in the case of 2/4 and 4/8) *and* gave a reason for their choice that was judged to be reasonable. The fraction pairs are presented in decreasing order of success.

The most straightforward pair (3/8, 7/8) and the most difficult pair (3/4, 7/9) were easily predicted in advance. Having said that, the percentage success on the easiest pair (77.1%), with *success* being defined as a correct choice coupled with an appropriate explanation**,** was not high. Given that students were interviewed at the end of their Grade 6 year, after probably some years of introductory work on fractions, nearly one-quarter of students do not seem to have a basic, part-whole understanding of fractions.

The vast majority (94.8% of successful students) noted that the denominator was the same (and hence the size of the parts), and therefore compared the numerators. However, 5.2% benchmarked to 1/2 and 1. Also 38.5% of all incorrect solutions (for which 3/8 was chosen as the larger) gave an explanation to the effect that "smaller numbers mean bigger fractions".

Table 1
The Percentage of Grade 6 Students Choosing Appropriately from Fraction Pairs With Appropriate Explanation (n = 323)

Fraction pair		*% correct*
3/8	7/8	77.1%
2/4	4/8	64.4%
1/2	5/8	59.4%
2/4	4/2	50.5%
4/7	4/5	37.2%
3/7	5/8	20.4%
5/6	7/8	14.9%
3/4	7/9	10.8%

The most difficult pair (3/4 & 7/9) proved to be very difficult for various groups of primary and junior secondary *teachers* with whom we have worked in professional development settings. Many teachers have been unable to offer an explanation beyond the use of common denominators, and so the 10.8% success rate for students is probably not surprising. In fact, 54.3% of successful students used common denominators and a total of 40% used some form of *residual strategy* (either 2/8 > 2/9 or 1/4 > 2/9 with some other explanation), whereas 5.7% (two students) converted the fractions to decimals in their heads.

The relative difficulty of the pair (4/7, 4/5) was a surprise to us, with only a 37.2% success rate, indicating that it was more difficult than (1/2, 5/8) and (2/4, 4/2). We did note however that 60.0% of all successful students provided an explanation similar to "there are four pieces in each, but as sevenths are smaller than fifths, so 4/5 will be larger", indicating the most common correct response was a strategy involving number sense rather than procedure. It was of some concern that 20.0% felt the need to convert to common denominators; 9.1% of successful students used benchmarking and 10.8% used residual thinking. This was a task in which *gap thinking* (Pearn & Stephens, 2004) was common, with 21.4% of students who chose 4/5 as larger providing inappropriate gap thinking reasoning (focusing on the difference between 4 and 7 and between 4 and 5). For all students who chose 4/7 as larger, 73.5% of reasons were to do with "larger numbers".

Benchmarking and residual strategies are a couple of the strategies that appear to be used by students displaying a more conceptual understanding of the size of fractions, yet they are not in widespread use by students or teachers in our schools. These strategies would have been most appropriate for the pairs (3/7, 5/8) and (5/6, 7/8) respectively, but the success rates were 20.4% and 14.9%. Of the successful students, 28.8% and 45.8% of students chose to use common denominators for these pairs respectively, thereby choosing a procedure rather than a strategy based more clearly on number sense. Also, 21.2% of all students used gap thinking for (3/7, 5/8) and 29.4% of all students claimed 5/6 and 7/8 were the same, often using gap thinking as their justification.

Student understanding of simple equivalences, appears to contribute to the relative success rate for the pairs (1/2 and 5/8, and 2/4 and 4/8) as most could identify ½ and 4/8 as the same, however, it must be said that 59.4% and 64.4% respectively are still lower than we predicted for students at the end of Grade 6.

The lack of emphasis on improper fractions in primary grades may account for the difficulty in explaining the relative size of 2/4 and 4/2 (42.7%). Also, the language some students use to label fractions may hinder their understanding. For example, some students were noted to read these as "two out of four" and "four out of two", which is not helpful when considering their respective size. "Two-quarters" and "four-halves", on the other hand, may help to create an image about the size of the parts that is more likely to lead to a correct solution.

Fractions as Division

8. *Pizza task.* Children were shown a picture (Figure 10), and told, "three pizzas were shared equally between five girls. … How much does each girl get?" Students were invited to use pen-and-paper if they appeared to require it.

Although 30.3% of Grade 6 students responded with a correct answer, it was apparent that most either drew a picture or mentally divided the pizzas to calculate the equal share. A concerning result was that 11.8% of students were unable to make a start. Greater exposure to division problems and explicit discussion connecting division with their fractional answers, for example, $3 \div 5 = 3/5$ may help lead students to the generalisation that $a \div b = a/b$.

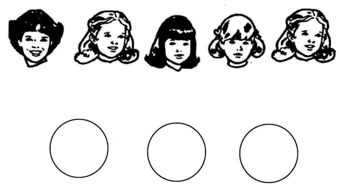

Figure 10. Pizza task.

Discussion

Despite the strong recommendations from researchers that school mathematics should provide students experiences with all key sub-constructs of fractions and the many useful models that illustrate these sub-constructs (Lamon, 1999; Post et al., 1993), it is clear that a large, representative group of Victorian Grade 6 students do not generally have a confident understanding of these and their use.

Generally, performance on *part-whole* tasks was reasonable, although when the object of consideration was not in a standard form and not broken into equal parts (e.g., the Fraction Pie task), less than half of the students could give a correct fraction name to the part. The teaching implications here are clear. Students need more opportunities to solve problems where not all parts are of the same area and shape. On the other hand, the dots array task showed that students handled this discrete situation well, *unitising* appropriately, and usually had access to fractions that were equivalent to a given fraction.

Although simple fraction as an *operator* tasks were straightforward for most students, it seems that only around one-sixth of students being able to find one-third of a half indicates that students may need more encouragement to form mental pictures when doing such calculations. The second part of the Fraction Pie task was closely related, and it is interesting that of the 138 students who solved the pie task correctly, only 47 could give an answer to "one-third of a half". On the other hand, of the 59 students who were successful with the mental task, 47 could solve the related pie task. Once again, the importance of visual images in solving such problems is clear.

The experience of the authors is that Australian students spend relatively little time working with number lines in comparison to countries such as The Netherlands. Given that only around half of the students could draw an appropriate number line that showed 2/3, it is clear that fraction as a *measure* requires greater emphasis in curriculum documents and professional development programs, as many students are clearly not viewing fractions as numbers in their own right. In light of these data, the performance on locating six thirds and eleven sixths was relatively high. The Construct a Sum and fraction comparison tasks revealed similar difficulties with understanding the size of fractions, particularly improper fractions, and a lack of use of *benchmarks* in student thinking. Emphasising these aspects instead of fraction algorithms may be wise.

From our experience, few Australian primary school and middle school teachers and even fewer students at these levels are aware of the notion of *fraction as division*. Most students who concluded that 3 pizzas shared between 5 people would result in 3/5 of a pizza each, either drew a picture or mentally divided the pizzas to calculate the equal share. A very small percentage knew the relationship automatically. This supports the data of Thomas (2002) that 47% of 14 year-olds thought 6÷7 and 6/7 were not equivalent.

In summary, our data indicate clearly that Victorian students (and probably their teachers through appropriate professional development) need greater exposure to the sub-constructs of fractions and the related models, as noted by Post et al. (1993) and other scholars. We would also encourage teachers to use some of the tasks we have discussed in one-to-one interviews with their students, as our experience is that the use of the interview provides teachers with considerable insights into student understanding and common misconceptions, and forms a basis for discussing the "big ideas" of mathematics and curriculum implications of what they have observed.

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